Dispersion relation of high-frequency plasma oscillations in Hall thrusters

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The dispersion relation of high-frequency plasma oscillations (>1 MHz) in a Hall thruster is estimated from two point probe measurements. The probes are located outside the accelerating channel nearby the channel exit. The probe orientation allows us to investigate the evolution of azimuthal and axial wave numbers with the oscillation frequency. The azimuthal dispersion relation is nearly linear. The axial dispersion relation depends upon the probe position, which translates into a varying slope. The observed features of the two dispersion relations can be explained in terms of spatial structure of the high-frequency plasma instabilities. *© 2008 American Institute of Physics*. [DOI: 10.1063/1.2889424]

The Hall thruster is a space propulsion technology that was initially developed in the 1970s in the frame of the Soviet space program.[1,2] It is nowadays frequently used onboard geostationary satellites for North-South and East-West station-keeping.[3] It also serves as the main propulsion system for interplanetary missions.[4] Basically, a Hall thruster is a plasma source with closed electron drift in crossed electric and magnetic fields. The magnitude of the magnetic field leads to an electron Larmor radius much smaller than any channel dimensions. On the contrary, the ion Larmor radius is very large. The low electron mobility across the magnetic field results in a localized axial voltage drop of the order of the discharge voltage U_d. Typical values of plasma parameters in a 1.5 kW-class Hall thruster are: an axial electric field $E_z \sim 10^4$ V/m, a radial magnetic field $B_r \sim 20$ mT, an electron temperature $T_e > 10$ eV, a charged particle density (under quasineutrality condition) $n_i \approx n_e \sim 10^{17}$ m^{-3} for a gas mass flow rate of $\dot{m}_a \sim 5$ mg/s, and an applied voltage of 300 V. The thrust corresponds to the momentum delivered to the escaping ion flow. A detailed description of Hall thrusters can be found in Refs. 1 and 2.

Numerous discharge instabilities with frequencies ranging from the kHz to the GHz domain, are observed in Hall thrusters.[1,2,5] Among these instabilities, the high-frequency (HF) instability (its lower limit varies according to different references $f \sim 1$–5 MHz) was shown to exist in the discharge of Hall thrusters in various theoretical works,[6–8] and it was experimentally identified as azimuthal wave with phase velocity close to the electron drift velocity $v_d = E_z/B_r$ in Hall thrusters of different design.[6,9–13] The HF instability is indeed azimuthally localized, however, there is some controversy about its axial propagation and its spatial expanse outside the accelerating channel.[6,9–11] Theoretical descriptions based on two-fluid models consider distabilizing effects associated with electron collisions and cross-field density gradients; they predict azimuthally propagating and axially localized waves.[6–8] In view of the covered frequency range, many authors suggested the existence of a relationship between HF instabilities and the electron dynamics, hence a profound interest in this type of instabilities despite the fact that they do not carry much energy contrary to e.g., "contour" oscillations (1–30 kHz) or "ion transit-time" oscillations (70–500 kHz).[1,2,5]

In this contribution we present and discuss azimuthal and axial dispersion relations of HF instabilities obtained experimentally in the plasma of the 5 kW-class PPSR®X000 Hall thruster[14] operated in the PIVOINE test bench.[15] The presented dispersion relations were obtained from a large dataset covering numerous operating conditions that warrants a high level of reliability.

The HF instability was studied by means of shielded Langmuir probes[12] located around the channel external wall away from the ion beam. The active part of a probe is made of 0.125 mm in diameter Ta wire and it has a length of ~6 mm. To access the azimuthal properties of the instability, two probes (P1 and P2) were placed in the channel exit plane with a fixed distance of 1.1 cm between their tips (Fig. 1). In spite of a possible small deviation from the azimuthal symmetry,[11–13] we consider the data obtained with this fixed probe pair reliable to assess an azimuthal dispersion relation $f = f(k_\theta)$. In order to access the axial properties of the instability and to derive an axial dispersion relation $f = f(k_z)$, two probes P3 and P4 were aligned along the thruster axis with a constant 1 cm gap [Fig. 1(b)]. In this configuration the probe pair could be moved in axial and radial direction. Measurements in azimuthal and axial directions were carried out separately and with different thruster operating parameters. Note that the probe configuration limits the investigation area; instability with a complex geometry would not be described in a proper way.

The probe signals are transferred through 8 m in length coaxial lines and they are recorded with a 4-channel Tektronix 5104B oscilloscope in the ac 50 Ω mode. A typical recording length is 80 μs at a sampling rate of 1.25×10^9 (see Ref. 12). Each dispersion relation is built from two distinct data sets.

The method to assess the dispersion relation from two-point measurements is described in numerous works.[16–19] It

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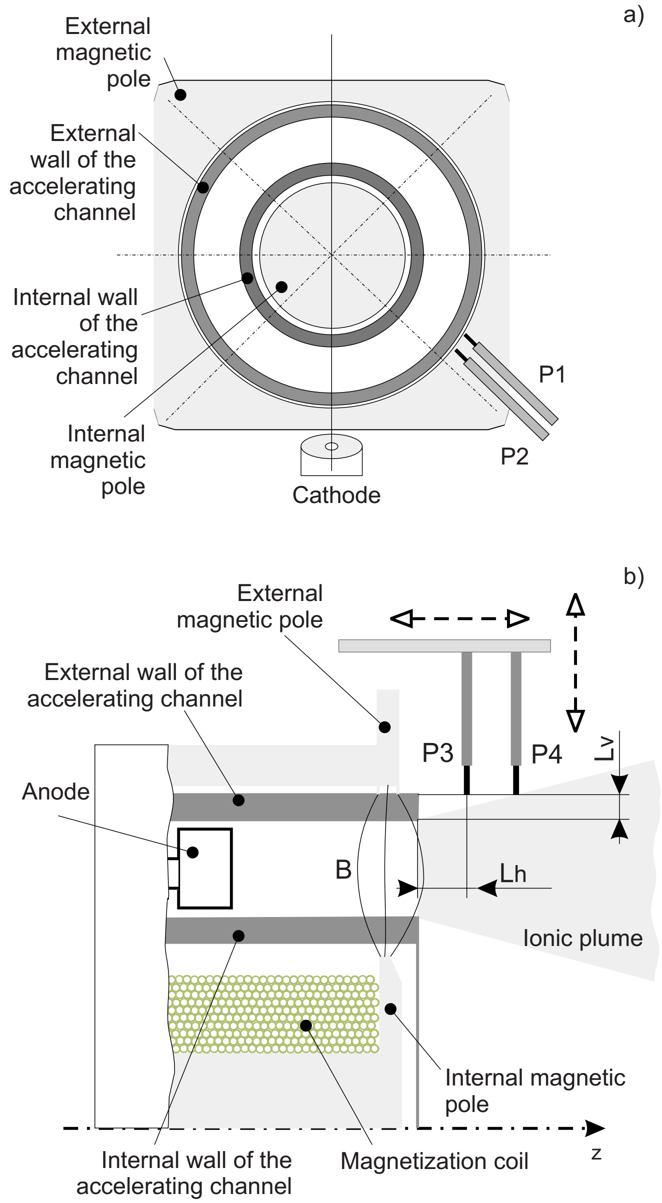

FIG. 1. (Color online) Probe positions on the PPSR®X000 thruster; (a) front view: for estimating azimuthal wavenumbers, $L_{P1-P2}=1.1$ cm; (b) side view (transversal cut): for estimating axial wavenumbers, $L_{P3-P4}=1$ cm; the magnetic field B geometry is shown schematically; the axial position L_h of the P3 tip relative to the channel cut-off and the radial position L_v relative to the channel external wall define the probe pair location.

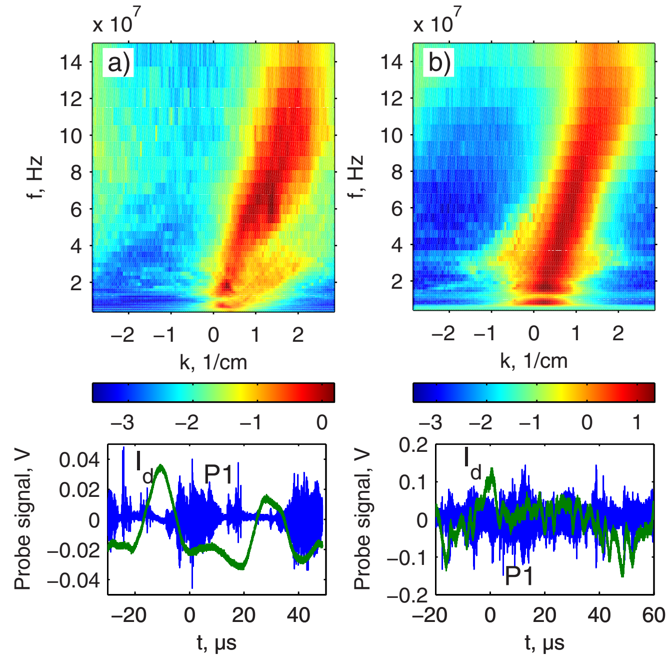

FIG. 2. (Color online) Azimuthal dispersion relations $f=f(k_\theta)$ (log 10 scale) and examples of probe signals and discharge current I_d waveform; (a) $\dot{m}_a=6$ mg/s, $U_d=350$ V, (b) $\dot{m}_a=6$ mg/s, $U_d=550$ V; the same value of magnetic field in both cases.

consists of the calculation of local wavenumber and frequency spectral density according to

$$S(k,\omega)=\langle 1/2[\Phi*(x,\omega)\Phi(x,\omega) + \Phi*(x+d,\omega)\Phi(x+d,\omega)]\delta(k_d-k)\rangle, \tag{1}$$

where $\Phi(x,\omega)=a(x,\omega)\exp[i\theta(x,\omega)]$ is the single probe frequency spectrum, d is the distance between the probes, and averaging is performed over the ensemble of measurements. The local wavenumber k_d is given by

$$k_d=[\theta(x+d,\omega)-\theta(x,\omega)]/d, \tag{2}$$

where $\theta(x,\omega)$ is the single probe signal phase spectral density. In order to avoid the conditions of planarity, statistical stationarity and homogeneity we follow the method described in Ref. 18 that consists of wavefield decomposition into plane wave packets by way of wavelet transform. Morlet wavelets are used in this paper.[18,19]

The azimuthal dispersion relation $f=f(k_\theta)$ was found to be nearly linear (Fig. 2). The azimuthal propagation velocity increases with discharge voltage, in agreement with earlier studies,[6,9–11,13] varying from 3.5×10^6 m/s for the low voltage operation mode [$U_d=350$ V, Fig. 2(a)] to 4.2×10^6 m/s for the high voltage case [$U_d=550$, Fig. 2(b)]. This velocity is close to the electron drift velocity $v_d=E_z/B_r$ and its direction coincides with the direction of the electron drift [clockwise in Fig. 1(a)]. However, a perfect match between the two velocities is arguable.[10] The comparison is further complicated by uncertainty on the location of the instability physical source as well as by the nonuniform distribution of plasma parameters and magnetic field.[5,20,21]

The cylindrical geometry of the system and the linear character of the azimuthal dispersion relation suggest the presence of multiple azimuthal modes $m=1,2,3,4\ldots$ and wavenumbers $k=m/R$, are in agreement with earlier studies.[5,6,9–11,13] Modes with k_θ varying from ~0.13 cm^{-1}, which corresponds to $m=1$, up to ~3 cm^{-1} were confidently identified.

A low-frequency branch exists in the dispersion relation plot in Fig. 2(a) (bottom-right corner), though it is less pronounced in Fig. 2(b). It originates in the HF instability development in the form of bursts of variable duration, as can

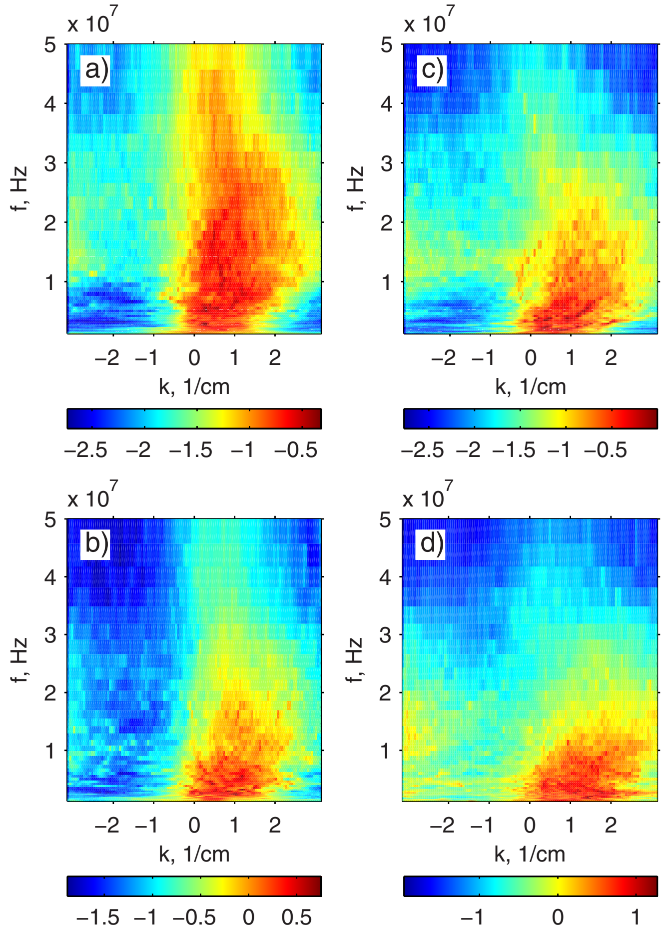

FIG. 3. (Color online) Axial dispersion relations $f = f(k_z)$ (log 10 scale); (a) $L_h = 5$ mm, $L_v = 10$ mm, (b) $L_h = 5$ mm, $L_v = 5$ mm, (c) $L_h = 10$ mm, $L_v = 10$ mm, (d) $L_h = 10$ mm, $L_v = 5$ mm ($\dot{m}_a = 8.3$ mg/s, $U_d = 550$ V).

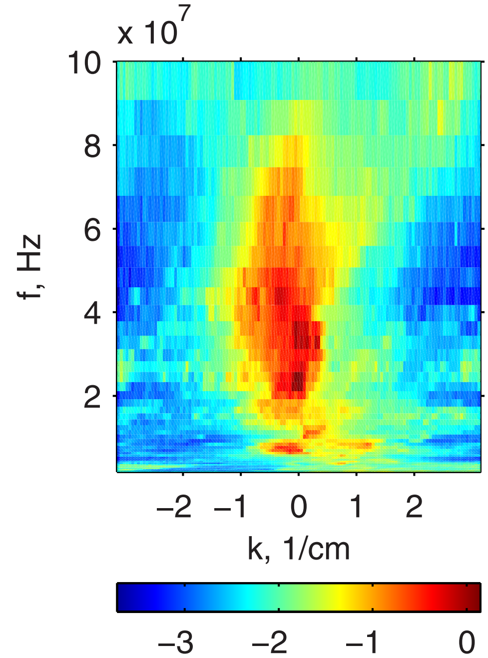

FIG. 4. (Color online) Axial dispersion relations $f = f(k_z)$ (log 10 scale), vertical slope ($\dot{m}_a = 5$ mg/s, $U_d = 500$ V, probes' location $L_h = 3$ mm, $L_v = 5$ mm).

be seen in Fig. 2(a). Inside each burst the HF instability azimuthal velocity varies slowly while from one series to another it can vary significantly. As was shown in earlier works,[9,11–13] such a well-defined pattern depends on thruster operating condition, especially magnetic field and discharge voltage, and it is correlated with discharge current low-frequency oscillations (20–40 kHz). When varying thruster operating conditions, the HF instability pattern changes from a series of bursts, which sometimes merges into a continuous time series as shown in Fig. 2(b), to a completely random configuration with no correlation between probes. In that case, it is impossible to extract any meaningful information from the dispersion relation. A detailed study that takes into account the bursty character of the HF events is underway to extract time and frequency resolved propagation properties.

Examples of the axial dispersion relation $f = f(k_z)$ at various locations under identical thruster operation conditions are shown in Fig. 3: 1) $L_h = 5$ mm, $L_v = 5$ mm, 2) $L_h = 10$ mm, $L_v = 5$ mm, 3) $L_h = 5$ mm, $L_v = 10$ mm, and 4) $L_h = 10$ mm, $L_v = 10$ mm [see Fig. 1(b)]. With this data set the slope of the dispersion relation is always steeper close to the channel exhaust [Figs. 3(a) and 3(b)]. A straightforward calculation of the axial phase velocity gives $(1-2) \times 10^5$ m/s close to the channel exit and $(5-6) \times 10^4$ m/s farther downstream; the two velocity vectors are directed towards the plume. The mean wavenumber k_z is about 0.8 cm^{-1}, which corresponds roughly to a wavelength of 8 cm. However, authors in Ref. 10 suggest from their own observations that the HF instability outside the accelerating channel is purely azimuthal. The HF instability was also found to have no axial phase shift in the accelerating channel.[11] While the slope of the azimuthal dispersion relation increases regularly with the discharge voltage, the slope of the axial dispersion relation depends in a more complex way on thruster operating parameters (mass-flow rate, discharge voltage, and magnetic field). A nearly vertical axial dispersion relation (see Fig. 4), corresponding to a constant axial phase shift for all frequencies, was observed whatever the probe axial position. Besides, a "tilted" relation was also observed for all probe positions [similar to Fig. 3(d)]. The signal amplitude increases slightly with the axial distance and it decreases when moving radially away from the thruster axis, as shown in Fig. 3.

The nonlinear shape and the position dependence of the axial dispersion relation can be interpreted as a specific HF instability spatial configuration. An instability extended in the axial direction, perpendicular to the magnetic field but tilted with respect to the electric field, and with predominantly an azimuthal velocity, could be responsible for the measured dispersion relations. The wave front of this tilted instability, propagating in the azimuthal direction with a velocity close to the electron drift velocity, reaches the axially separated probes at different moments in time, hence producing a phase-shift. The instability inclination angle, as determined by the respective values of k_θ and k_z, depends on the thruster operating parameters and can also vary along the instability front. Such HF instability spatial configurations are more general than the strictly axially aligned one reported in Ref. 10. The axial extension of HF instability from the channel side was shown to be limited by the maximum of stationary magnetic field.[11] The instability maximum extension into the plume was not confidently identified, as the probes were not exposed to the ionic plume. Nonetheless, as previously mentioned, 2 cm away from the channel exit plane, the HF instability is still strong.

This interpretation is in agreement with recent measurements of turbulent magnetic field in Hall thrusters, which exhibits a periodic pattern identical to the one detected by HF probes.[22] It was shown that the source of the HF magnetic field can be modeled by a collection of currents of charged particles, localized in the plasma volume and rotating in azimuth with the velocity of HF instability. The HF instability can be strong enough to insure an effective collision frequency sufficient for the anomalous turbulent electron transport in Hall thrusters.[22] Moreover, the charged particle currents could represent an electron anomalous current through the magnetic barrier.[22] Therefore, all results are in favor of the existence of spatially organized, azimuthally rotating, structures in the very near-field of Hall thruster plasma. Such structures can be the zones of anomalous electron transport. These results also highlight the two-dimensional (at least) character of HF instabilities, which is an important property for the theoretical works as well as numerical simulations.

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