Observation of high-frequency ion instabilities in a cross-field plasma

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Abstract
This paper reports on the examination of the high-frequency ion dynamics in a low-pressure cross-field plasma. Measurements of the time-varying ion velocity distribution function were carried out by photon-counting laser-induced fluorescence spectroscopy. A temporal correlation was found between the low-frequency ionization process and the high frequency ion instability. The high-frequency spectral properties of the ion mean velocity are compared with a perturbed fluid model of the plasma flow on the source axis. The wavelength of the high-frequency mode is similar to the electric field extent outside the cavity. Surprisingly the mode appears only in the area of negative magnetic field gradient.

Keywords: Hall thruster, laser-induced fluorescence, instability

(Some figures may appear in colour only in the online journal)
discharge generated between an cylindrical coordinate system. Have been recorded along the discharge current at a low-frequency resonance originating from the ionization process, and referred to as the breathing mode [5]. A coherent behavior of the discharge has been obtained at a modulation frequency of 14 kHz. Synchronizing the discharge modulation and the acquisitions allows for averaging over a very large number of cycles, which ensures a high signal-to-noise ratio. Notice that the modulation of the plasma breathing oscillations has been shown not to perturb the time-averaged behavior of the discharge, unlike previous techniques that relied on periodic and short discharge power cuts [9, 10]. This diagnostic technique has been successfully used to observe the low-frequency oscillations of the electric field that is established in these plasmas [11].

The source used in this study is a Hall thruster, used for in-space propulsion of satellites and exploration probes. The source has been described in a previous work [12]. It consists in a low-pressure discharge generated between an anode, placed at the upstream end of an annular ceramic cavity, and an external hollow cathode. The magnetic field is generated by SmCo magnets. The typical field strength is around a hundred Gauss, so that the electrons are magnetized while the more massive ions are not. The discharge voltage is 200 V. Xe has been used as a working gas. The anode mass flow rate is 1 mg s$^{-1}$ and the cathode mass flow rate is 0.2 mg s$^{-1}$. The source is electrically floating, which mimics the flight mode operation. Axial IVDFs, $f(x, v_z, t)$, have been recorded along the source channel axis, from -13 mm up to 10 mm, where the position 0 mm is the cavity exit plane. Due to the annular geometry of the source, we use a $(r, \theta, z)$ cylindrical coordinate system.

Figure 1 shows the raw IVDF recorded at $z = 3$ mm away from the source cavity for frequencies below 1 MHz, and illustrates the measurement outcome. The number of fluorescence photons is acquired as a function of both the ion velocity and time during a breathing oscillation. Each ion velocity group has been probed during approximately 5 min, through 176 000 acquisitions on 7150 bins of the counting card.

![Figure 1](Image)

**Figure 1.** High frequency oscillations of the IVDF at 3 mm downstream the exit plane during a breathing oscillation. The colorbar indicates the number of fluorescence photons. Each ion velocity group has been probed during approximately 5 min, through 176 000 acquisitions on 7150 bins of the counting card.

Temporal resolution is 100 ns. Fifteen wavelengths that best describe the time-averaged IVDF are chosen to yield the time-dependent IVDF. Time coherence is ensured by modulating the discharge current at a low-frequency resonance originating from the ionization process, and referred to as the breathing mode [5]. A coherent behavior of the discharge has been obtained at a modulation frequency of 14 kHz. Synchronizing the discharge modulation and the acquisitions allows for averaging over a very large number of cycles, which ensures a high signal-to-noise ratio. Notice that the modulation of the plasma breathing oscillations has been shown not to perturb the time-averaged behavior of the discharge, unlike previous techniques that relied on periodic and short discharge power cuts [9, 10]. This diagnostic technique has been successfully used to observe the low-frequency oscillations of the electric field that is established in these plasmas [11].

The Hilbert–Huang spectral analysis has been applied to the measured time series [13, 14]. Unlike Fourier transform, this technique is well-suited for the study of non-stationary and non-linear signals. Based on the Huang sifting algorithm, the Hilbert spectral method allows the computation of the Hilbert and frequency spectra of a signal. The frequency spectrum is called a marginal spectrum in the Hilbert spectral technique [13, 14]. These spectra are somewhat similar to a Fourier spectrum, with the additional contribution of the time-dependent nature of the frequency content.

The spectral analysis outcome is exemplified in figure 2. The marginal spectrum of the ion mean velocity at 1 mm downstream the source exit plane is shown. Evidence of low frequency IVDF oscillations can be found elsewhere [11]. Oscillations of the IVDF at a few hundreds of kHz are revealed for the first time. Low- and high-frequency oscillations of the IVDF have been observed at every measurement location.

**Figure 2.** Marginal and Hilbert spectra of the ion mean velocity oscillation at $z = 1$ mm: (top) marginal spectrum; (bottom) oscillation at 575 kHz. The colorbar indicates the normalized power level.
While the low frequency oscillations are well-understood, the transit time instability description has not progressed much from an experimental viewpoint since the early work of Esipchuk and Morozov in 1973 [15], rendering the comparison with fluid models impossible. Observations of the floating plasma potential reported at that time an instability, occurring at a frequency corresponding to the reciprocal ion time-of-flight through the acceleration zone, \( f = v_i/L \), with a wide spread in frequency so that \( \Delta f \approx f \).

The marginal spectrum of the time-resolved ion mean velocity has been computed at each measurement location, which allows to detect the existence of the instability, and ultimately to compute its frequency content. In order to extract more information from the measured IVDF, we correlate the experimental observations to a linearised one-dimensional two-fluid model of the plasma, and determine the plasma parameters which influence the instability.

The plasma is assumed to be composed of cold unmagnetized ions and cold magnetized electrons. The magnetic field is assumed to be purely radial, which is a reasonable approximation on the cavity axis. We study the case of quasineutral ions and cold magnetized electrons. The magnetic field parameters which influence the instability.

\( \omega = \omega_0 + i\gamma \) where \( \omega_0 \) is the mode frequency and \( \gamma \) is the growth rate of the instability.

\[
\omega = v_ik - \frac{\omega_n \omega_m}{v_e^2} \left[ \frac{ik}{v_e^2} + \left( \frac{1}{v_e^2} - \frac{1}{v_i^2} \right) - ik \right]^{1/2}, \tag{3}
\]

The growth rate of the instability is mapped in figure 3, in MHz, as a function of the mode wavenumber and axial position along the cavity axis. Regions in which the growth rate is positive, and where unstable modes are therefore likely to appear, extend downstream the source exit plane, for \( z > 1 \) mm and \( k < 1000 \) m\(^{-1}\). The corresponding wavelength is on the order of the width of the electric field distribution outside the cavity. The growth rate becomes negative for higher wavenumbers between \( z = 4\text{–}9 \) mm, which is the location of the high electric field region. Details concerning the electric field distribution in this source can be found in [11]. It is worth noting that the growth rate is negative for any wavenumber in regions where \( z < 1 \) mm, i.e. inside the cavity. Expanding the dispersion relation and factoring out the powers of \( \gamma \), it can be seen that, in our operating conditions, the sign of the growth rate is determined by the opposite sign of \( \frac{\omega_n \omega_m}{v_e^2} \) to that of \( \frac{1}{v_e^2} - \frac{1}{v_i^2} \). Owing to the strong axial electric field, \( \frac{\partial \Phi}{\partial z} > 0 \) at each location, and therefore the instability triggering is primarily determined by the magnetic field profile.

The mode frequencies are mapped in figure 4, in MHz, in the area of positive growth rate, as a function of the mode wavenumber and axial position along the cavity axis. The instability is more likely to occur at low wavenumbers, which scale with the acceleration zone length, on the order of 1 cm. In this region, frequencies range from 150 kHz to 1.2 MHz,
A noticeable feature can be found in the function of frequency scale: the frequency spread is on the same order of magnitude as the instability first reported by Esipchuk and Morozov [15], i.e. the experimental results confirm the turbulent nature of the 1.1 MHz band. Notice that similar, ranging in the 100 kHz—of magnitude of the mode frequency indeed appears to be expressed in arbitrary units. Agreement can be found between the axial position along the cavity axis. The colorbar is temporal evolution are presented in figure 5, as a function of-flight through the device which is on the order of magnitude of the reciprocal ion time-

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**References**